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Tatjana Stadnitski ([tatjana.stadnitski@uni-ulm.de](mailto:tatjana.stadnitski@uni-ulm.de))

**ISSN** 2280-3769 (online)

**Article type:** Commentary

**Submission date:** 11 October 2012

**Acceptance date:** 21 June 2013

**Publication date:** 21 June 2013

**Article URL:** <http://www.fractal-lab.org/articles/2013/1.pdf>

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# Detecting Fractal Behavior

Tatjana Stadnitski

Research Associate, University of Ulm

Correspondence should be addressed to:

Tatjana Stadnitski,  
Department of Psychology  
University of Ulm  
Albert-Einstein-Allee 47  
89081 Ulm  
Germany  
(E-mail: [tatjana.stadnitski@uni-ulm.de](mailto:tatjana.stadnitski@uni-ulm.de)).

## Detecting Fractal Behavior

Detecting fractal behavior over time is a sophisticated methodological issue for the following reasons. There are various mathematical approaches with their respective statistical parameters that measure fractality. For each parameter, numerous estimators of different quality are available and none of the procedures is clearly superior to the other [1; 2; 3]. Statistical characteristics of some non-fractal empirical time series can resemble fractal patterns, which complicates the identification. Therefore, the method that is chosen influences the research outcomes [4]. Detecting fractals in the cognitive processes confronts some additional challenges. Unlike biological or physical measurements, intentional activity is deliberate, and thus easy to manipulate. Hundreds of observations are necessary for detecting behavioral fractal patterns, hence it is possible that, due to fatigue or inability to fulfill the task properly, some reactions do not originate from the assumed cognitive mechanism but represent just random responses [5]. The present article is a brief commentary of the above mentioned topics.

### Fractal Parameters

There are different mathematical devices to express self-similar power-law organization of fractal structures. Since fractal signals can be analyzed in both time and frequency domains, there exist different fractal parameters like the Hurst coefficient ( $H$ ), the scaling exponent  $\alpha$ , the power exponent  $\beta$  of the spectral analysis, or the differencing statistic  $d$  of the ARFIMA (Autoregressive Fractionally Integrated Moving Average) framework. It is crucial to understand that these parameters express exactly the same characteristics, which implies that each quantity can be converted to the other. For instance, interrelations between the parameters in stationary time series are:  $\beta=2d$ ,  $H=(\beta+1)/2$ , and  $\alpha=H=d+0.5$ . The expected theoretical parameter values of the  $1/f$  noise are  $d=0.5$ ,  $\beta=1$ ,  $\alpha=H=1$  [4].

## The Choice of the Measurement Method

The choice of the measurement method determines the outcome of fractal analyses for the following reasons. Several estimators of fractal parameters are available (Table 1). Due to their complexity, it is not possible to compare them mathematically. Therefore their properties (e.g., bias, variability) are examined by means of a Monte Carlo method. Evaluation studies revealed that none of the procedures is superior to the other. The performance of the methods strongly depends on the aspects like the properties of the underlying process (e.g., stationary vs. non-stationary) or empirical context [2; 3; 6; 7]. For instance, Maximum Likelihood algorithms, the most accurate estimation techniques of the time domain, can handle only stationary data [1]. Most estimators from the frequency domain can be applied directly to stationary and non-stationary time series, however, they are less precise than ML methods. Moreover, they tend to fail in empirical series combining fractal and non-fractal dynamics [3; 5]. Hence, the most important finding from simulation studies is that fractal estimators can produce erroneous results under disadvantageous conditions. Further, estimates from the same time series obtained from different methods can vary markedly. Consequently, comparisons of results from studies in which fractality were determined with different measurement instruments are problematic and must be interpreted cautiously.

The fact that knowledge about statistical properties of fractal estimators originate from Monte Carlo experiments underlines the importance of accurate data simulation. Obviously, an exact generation of fractal structures is a necessary condition for correct inference from Monte Carlo studies. There is some empirical evidence that even popular simulation algorithms of fractal noise like *lmSimulate* of R can be inaccurate, therefore data generation techniques must always be questioned and evaluated [5]. If possible, Monte Carlo results should be cross validated by employing several procedures.

## Measurement Artifacts in Cognitive Performances

Fractals have been observed in different domains of cognitive psychology including visual perception [8], controlled behavior [9], and automatic performances such as word naming or simple reaction times [10]. For instance, Gilden and his colleagues discovered  $1/f$  noise in experiments including mental rotation, lexical decision, shape and color discrimination, or visual search [11; 12]. In this research paradigm, participants are required to complete hundreds of related tasks and press a key each time they detected a stimuli or a change. Response times between stimulus presentation and reaction serve as dependent measures. Fractal analyses are then applied to the response times data.

The described operationalization of cognitive performances makes a reliable identification of fractal structures especially challenging. The problem is that the dependent measure combines cognitive and motor components. Moreover, hundreds of trials are necessary to detect fractal organization of cognitive processes. Thus, participants of such experiments complete a vast number of monotonous tasks, which can be rather tiring. As a result stochastic trends in reaction times are possible due to fluctuations in attention or fatigue. Inability to fulfill the task properly can lead to random responses introducing white noise into the data. The interplay of these factors can imitate fractal structures. For instance, it is known that a combination of non-fractal signals like white noise and random walk can produce patterns similar to  $1/f$  noise [13]. Therefore, fractal methods used with these processes must be able to discern fractal behavior in the presence of non-fractal noise and to reliably distinguish between genuine fractals and fractal like signals.

## Implication for Applied Research

The discussed topics implicate the following practical recommendations for applied researchers investigating fractal behavior in cognitive processes. First of all, it is important to know different fractal methods. Comparison of strengths and constraints of diverse algorithms

can help to identify the procedure that works best under specific research conditions. Due to the fact that no method outperforms the other in a majority of empirical situations, a strategic approach is necessary for proper measurement of fractal parameters. An example of such strategy and a detailed description of its implementation in applied settings are available in [3] and [4]. Finally, it is necessary to account for the deliberateness of cognitive activity. In contrast to biological measurements, cognitive responses are easy to manipulate. Monitoring of erroneous trials can help to determine whether observed reactions originate from the assumed problem solving mechanism or represent just random responses.

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Table 1. *Estimators of Fractal Parameters Freely Available in the Software Package R.*

| Procedure                        | Outputs estimates of | Available               | Evaluated |
|----------------------------------|----------------------|-------------------------|-----------|
| DFA                              | $\alpha$             | library <i>fractal</i>  | [2; 5;7]  |
| SSC                              | $\alpha$             | library <i>fracdiff</i> | [5; 6]    |
| <sup>low</sup> PSD               | $\beta$              | [4]                     | [5; 6; 7] |
| <sup>low</sup> PSD <sub>we</sub> | $\beta$              | [4]                     | [5; 6; 7] |
| hurstSpec                        | $\alpha$             | library <i>fractal</i>  | [2; 3; 5] |
| fdGPH                            | d                    | library <i>fracdiff</i> | [2; 3; 5] |
| fdSperio                         | d                    | library <i>fracdiff</i> | [2; 3; 5] |
| FDWhittle                        | d                    | library <i>fractal</i>  | [2; 3; 5] |
| fracdiff                         | d                    | library <i>fracdiff</i> | [2; 3; 5] |
| Approximate ML                   |                      |                         |           |